Determining the HBT base-collector elements directly from S-parameter data

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Abstract— **This paper gives suitable analytic equations for direct extraction of the heterojunction bipolar transistor (HBT) series base resistance R_{bi} and base-collector capacitances (C_{bci} and C_{bcx}), elements which determine the transistor maximum oscillation frequency f_{max}. New expressions, one for R_{bi}, and the other relating R_{bi} and C_{bcx} in terms of measured parameters have been developed. On this basis, direct extraction of the inner C_{bci} and outer C_{bcx} base-collector capacitances is possible, independently and robustly. All expressions are valid at each measured frequency, a useful feature in the control of extraction quality.**

I. Introduction

The high frequency performance of HBTs is determined significantly by the base-collector elements and in fact reducing the values of these elements has been a major driver for developing new HBT technologies such as transferred-substrate technology which minimizes the base-collector capacitance [1] and regrown base-contacts for minimizing the base resistance [2]. Uncertainty, however, still remains in the determination of the small-signal equivalent circuit elements for modeling this critical base-collector junction.

The base resistance R_{bi} is difficult to extract and several approximate formulae for extracting this element have recently been proposed [4], [5]. On the other hand, extraction of the total base-collector capacitance modeled by an outer and inner value, C_{bcx} and C_{bci}, respectively is readily done [6]. However, its separation in to the individual components relies on simplifying assumptions [3]. Recent work extracts the capacitance values using a least squares fit [4], and just like for the approximate formulae for R_{bi}, no checking of the quality of extraction on the basis of frequency (in)dependence can be done.

Recently, the authors proposed a set of analytical equations for extracting each of the intrinsic HBT elements, and in particular resolved C_{bcx} exactly in terms of measured parameters [7]. For the elements under discussion the earlier formulation lacks the robustness required for routine usage. In this paper, it is shown that the earlier results are not unique and in fact a host of other exact equations may be derived for extraction of R_{bi}, C_{bci} and C_{bcx}. It suffices to note here that any exact expression which has the output resistance R_{bc} being effectively in the numerator of the measured parameters is very sensitive to even very small variations in the measured data, and hence most such equations are only of limited value. It is on this basis that equations presented in this paper were finally selected, and they form a robust set for modeling the base-collector junction. Extraction results for InP/InGaAs HBTs are given.

II. Analysis

First of all, the pad capacitances, and the base and collector access impedances are determined and de-embedded from the measured data as outlined in [6]. The series emitter resistance and inductance need not be known before the determination of the base-collector elements. The resultant HBT equivalent circuit is as shown in Fig.1 and analysis of this circuit yields the following analytical equations involving R_{bi}, C_{bci} and C_{bcx} in terms of Z-parameters. Details of the derivation of these equations including their dual in Y-parameters are given in the Appendix.

$$R_{bi}Y_{bci} = \frac{Z_{11} - Z_{12}}{Z_{22} - Z_{21}} \tag{1}$$

$$Y_{bcx} = \frac{1}{\Sigma Z}\left(1 - \frac{Z_{11} - Z_{12}}{R_{bi}}\right) \tag{2}$$

where $\Sigma Z = Z_{11} - Z_{12} + Z_{22} - Z_{21}$, $Y_{bci} = 1/R_{bc} + j\omega C_{bci}$, and $Y_{bcx} = j\omega C_{bcx}$.

Note that (2) may be re-arranged in various ways to give exact extraction equations for R_{bi} and C_{bcx} by taking either the real or imaginary part of the re-arranged equation, for instance the real part of (2) gives R_{bi}.

Now consider the expression for the total base-collector capacitance given by (see (7) in the Appendix)

$$\omega\left(C_{bcx} + C_{bci} + \frac{R_{bi}}{R_{bc}}C_{bcx}\right) = \mathrm{Im}\left(\frac{1}{Z_{22} - Z_{21}}\right) \tag{3}$$

The left hand side of (3) essentially consists of the imaginary parts of (1) and (2) with the third term being practically insignificant. Using (1) and (2) in (3) and re-arranging gives the following equation for R_{bi}

$$R_{bi} = \frac{\mathrm{Im}\left\{\frac{Z_{11}-Z_{12}}{Z_{22}-Z_{21}} - \frac{Z_{11}-Z_{12}}{\Sigma Z}\right\}}{\mathrm{Im}\left\{\frac{1}{Z_{22}-Z_{21}} - \frac{1}{\Sigma Z}\right\}} \quad (4)$$

Once R_{bi} is known, C_{bci} and C_{bcx} are determined using (1) and (2).

It is important at this point to note that extraction attempts using the exact equations for R_{bi} or C_{bcx} arising from (2) on both synthetic and real transistor data showed that the equations were very sensitive to the accuracy of the parasitic elements and therefore are not suitable for extraction purposes. It seems that whenever R_{bc} appears effectively in the numerator of the measured parameters then the extraction is very sensitive to even small errors which may occur in determining the access impedances. Notice that for this reason, all equations (1)-(4) have R_{bc} effectively in the denominators of the measured data.

III. Results

Analysis of measured S-parameter data of a 4x10 InP/InGaAs HBT using (1)-(4) biased at $V_{CE} = 1.27$V and $I_C = 20$mA is presented in this section. Fig.2 illustrates the suitability of using (1) for the determination of the product of the base resistance R_{bi} with the inner base-collector capacitance C_{bci}. Fig.3(a) shows the extracted series base resistance using (4). And finally, Fig.4 shows the extracted inner, outer and total base-collector capacitances using (1) & (4), (2) & (4), and (3) respectively. All are essentially independent of frequency as would be expected of real/physical elements.

Varying either or both of the pad capacitances by up to ±20% is a simple test to extraction robustness; sensitive equations give totally different and frequency dependent extracted values. This is a simple and valid testing strategy since the intrinsic base-collector elements are being determined by analytic equations and therefore the accuracy of the extrinsic elements determines the overall modeling accuracy.

An alternative method of validating the results involves the direct extraction of the following quantity which arises from (1) and (3)

$$R_{bi}\frac{C_{bci}}{C_{bci}+C_{bcx}} = \frac{\mathrm{Im}\left(\frac{Z_{11}-Z_{12}}{Z_{22}-Z_{21}}\right)}{\mathrm{Im}\left(\frac{1}{Z_{22}-Z_{21}}\right)} \quad (5)$$

For a good extraction, the elements R_{bi}, C_{bci} and C_{bcx} as determined by the earlier expressions should conform to (5). Fig.3(b) in conjunction with all the other plots demonstrates this over all the measurement frequency range.

And finally, the extracted R_{bi} gives the best indication to the quality of extraction. A flat R_{bi} over the measurement frequency indicates an excellent extraction. Inaccuracy, however, in determining the access parasitics may lead to extracting some values which are clearly out of range (e.g. being negative) at some measurement points and so such values must be filtered out before averaging. Other elements are determined using [6].

IV. Conclusion

New equations for accurately modeling the base-collector equivalent circuit elements of an HBT have been presented. Limitations to other possible formulations have been pointed out. The results focussed on the physical relevance of the extracted elements and the extraction quality/robustness as may be demonstrated by every extracted element at each measurement frequency.

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Appendix

I. Analysis of the intrinsic HBT

This section gives a complete derivation of the extraction equations presented in this paper. The analysis is for the HBT equivalent circuit as shown in Fig.1 where parasitic capacitances and series parasitic elements (Z_B and Z_C) are known and de-embedded. Z_E need neither be known nor be de-embedded.

A. Formulation using Z-parameters

The Z-parameters of the equivalent circuit in Fig.1 are given below.

$$Z_{11} - Z_{12} = \frac{R_{bi} Z_{bcx}}{Z_{bci} + Z_{bcx} + R_{bi}} \tag{6}$$

$$Z_{22} - Z_{21} = \frac{Z_{bci} Z_{bcx}}{Z_{bci} + Z_{bcx} + R_{bi}} \tag{7}$$

$$Z_{12} - Z_{21} = \frac{\alpha Z_{bci} Z_{bcx}}{Z_{bci} + Z_{Z_{bcx}} + R_{bi}} \tag{8}$$

$$Z_{12} = Z_E + Z_{BE} + \frac{(1-\alpha) Z_{bci} Z_{bcx}}{Z_{bci} + Z_{bcx} + R_{bi}} \tag{9}$$

where Z_{bci} and Z_{bcx} are given by

$$Z_{bci} = \frac{R_{bc}}{1 + j\omega R_{bc} C_{bci}} \tag{10}$$

$$Z_{bcx} = \frac{1}{j\omega C_{bcx}} \tag{11}$$

Dividing (6) with (7) gives

$$R_{bi} Y_{bci} = \frac{Z_{11} - Z_{12}}{Z_{22} - Z_{21}} \tag{12}$$

and now taking the reciprocal of (6) gives

$$\frac{1}{Z_{11} - Z_{12}} = \frac{1}{R_{bi}} + Y_{bcx}\left(1 + \frac{1}{R_{bi} Y_{bci}}\right) \tag{13}$$

which on using (12) and a bit of rearrangement gives

$$Y_{bcx} = \frac{1}{\Sigma Z}\left(1 - \frac{Z_{11} - Z_{12}}{R_{bi}}\right) \tag{14}$$

with

$$\Sigma Z = Z_{11} - Z_{12} + Z_{22} - Z_{21} \tag{15}$$

(14) can be re-arranged in various ways to give exact extraction equations for R_{bi} and C_{bcx} by taking either the real or imaginary part of the re-arranged equation, e.g. taking its real part gives R_{bi}.

B. Formulation using Y-parameters

The Y-parameters of Fig.1 are given below. Here, Y_{be} includes Z_E without loss of exactness and generality

$$Y_{11} = Y_{bcx} + \frac{Y_{bci} + (1-\alpha) Y_{be}}{1 + R_{bi}\left[Y_{bci} + (1-\alpha) Y_{be}\right]} \tag{16}$$

$$Y_{12} = -Y_{bcx} - \frac{Y_{bci}}{1 + R_{bi}\left[Y_{bci} + (1-\alpha) Y_{be}\right]} \tag{17}$$

$$Y_{21} = -Y_{bcx} + \frac{-Y_{bci} + \alpha Y_{be}}{1 + R_{bi}\left[Y_{bci} + (1-\alpha) Y_{be}\right]} \tag{18}$$

$$Y_{22} = Y_{bcx} + \frac{Y_{bci}(1 + R_{bi} Y_{be})}{1 + R_{bi}\left[Y_{bci} + (1-\alpha) Y_{be}\right]} \tag{19}$$

From the above equations, it is easy to show that

$$R_{bi} Y_{bci} = \frac{Y_{22} + Y_{12}}{Y_{11} + Y_{21}} \tag{20}$$

and

$$\alpha = \frac{Y_{21} - Y_{12}}{Y_{11} + Y_{21}} \tag{21}$$

Re-writing (17) as

$$-(Y_{12} + Y_{bcx}) = \frac{Y_{bci}}{1 + R_{bi}\left[Y_{bci} + (1-\alpha) Y_{be}\right]} \tag{22}$$

and using (17) and (18) to give

$$Y_{21} - Y_{12} = \frac{\alpha Y_{be}}{1 + R_{bi}\left[Y_{bci} + (1-\alpha) Y_{be}\right]} \tag{23}$$

$$= \frac{1}{R_{bi}} \frac{\alpha \frac{Y_{be}}{Y_{bci}}}{\frac{1}{R_{bi} Y_{bci}} + \left[1 + (1-\alpha)\frac{Y_{be}}{Y_{bci}}\right]} \tag{24}$$

leads to

$$\frac{Y_{bci}}{Y_{be}} = -\alpha \frac{Y_{12} + Y_{bcx}}{Y_{21} - Y_{12}} \tag{25}$$

Now using (20) and (25) in (23), and after a few simple algebraic manipulations we obtain

$$\frac{1}{R_{bi}} = Y_{11} + Y_{12} - (Y_{12} + Y_{bcx})\left(1 + \frac{1}{R_{bi} Y_{bci}}\right) \tag{26}$$

$$= Y_{11} - Y_{12}\frac{Y_{11} + Y_{21}}{Y_{22} + Y_{12}} - Y_{bcx}\left(\frac{\Sigma Y}{Y_{22} + Y_{12}}\right) \tag{27}$$

which may be re-written as

$$Y_{bcx} = \frac{\Delta Y}{\Sigma Y} - \frac{Y_{22} + Y_{12}}{R_{bi} \Sigma Y} \tag{28}$$

with $\Delta Y = Y_{11} Y_{22} - Y_{21} Y_{12}$ and $\Sigma Y = Y_{11} + Y_{12} + Y_{21} + Y_{22}$. Here, (28) can also be re-arranged in various ways to give exact extraction equations for R_{bi} and C_{bcx} by taking either the real or imaginary part of the re-arranged equation. Using (20) and (28) in place of (1) and (2) leads to identical results.

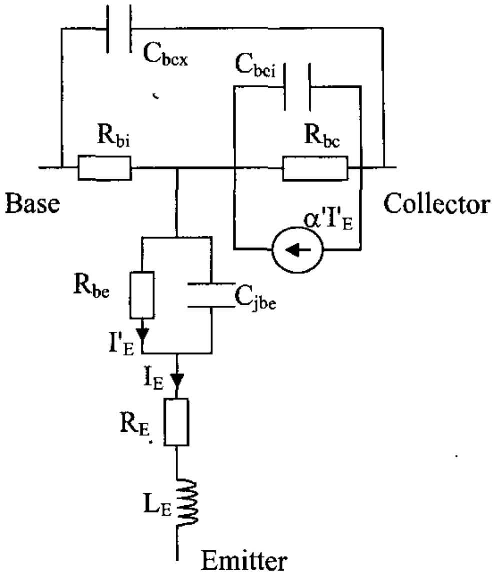

Fig. 1. HBT equivalent circuit with the base and collector access parasitic elements de-embedded. $\alpha' = \alpha_0 e^{-j\omega\tau_1} / (1 + j\omega\tau_b)$.

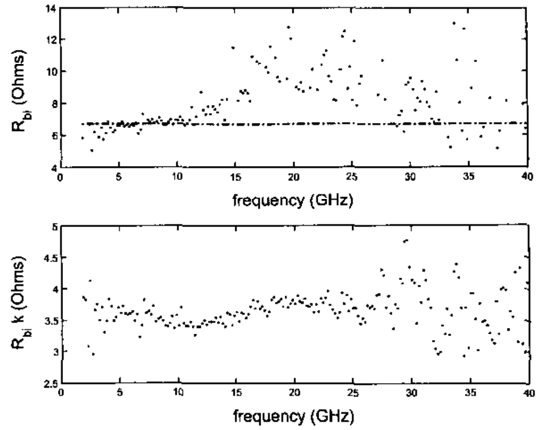

Fig. 3. a) Top plot: Plot of extracted R_{bi} versus frequency. In this particular case, R_{bi} is extracted over the lower frequency range (see dashed line). b) Bottom plot: Extracted $R_{bi}k$ (with $k = C_{bci}/(C_{bci}+C_{bcx})$) versus frequency. $V_{CE} = 1.27$V and $I_C = 20$mA.

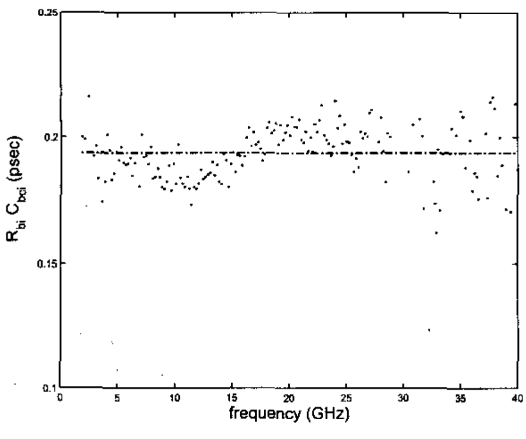

Fig. 2. Extracted $R_{bi}C_{bi}$ from the imaginary part of (1). Mean value is shown by dashed line. $V_{CE} = 1.27$V and $I_C = 20$mA.

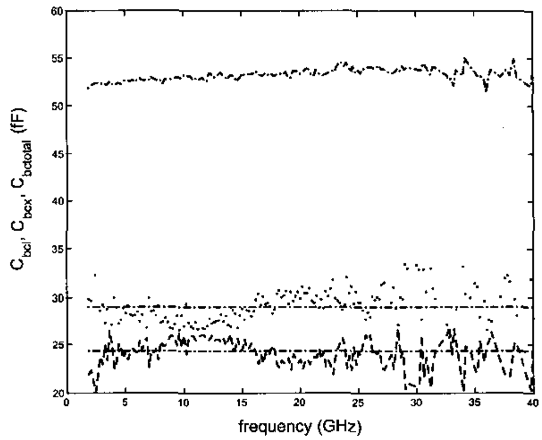

Fig. 4. Plot of C_{bci} (·), C_{bcx}(- -) and $C_{bci}+C_{bcx}$ (- ·) versus frequency. $V_{CE} = 1.27$V and $I_C = 20$mA.